

Math 10A HW9 Solutions

(1)

$$\begin{aligned}4a_{n-1} - 3a_{n-2} &= 4(2 \cdot 3^{n-1} + 1) - 3(2 \cdot 3^{n-2} + 1) \\&= (4 \cdot 2 \cdot 3^{n-1} + 4) - (2 \cdot 3^{n-1} + 3) \\&= (4 \cdot 2 \cdot 3^{n-1}) - (2 \cdot 3^{n-1}) + 4 - 3 \\&= \left(\frac{8}{3}\right) 3^n - \left(\frac{2}{3}\right) 3^n + 1 \\&= \left(\frac{6}{3}\right) 3^n + 1 \\&= 2 \cdot 3^n + 1 \\&= a_n \checkmark\end{aligned}$$

$$\begin{aligned}(2) \quad 2a_{n-1} + 1 &= 2(2^{n-1} - 1) + 1 \\&= 2^n - 2 + 1 \\&= 2^n - 1 \\&= a_n \checkmark\end{aligned}$$

$$\begin{aligned}\text{Also } a_1 &= 2a_0 + 1 = 2(0) + 1 = 1 \\a_1 &= 2^1 - 1 = 1.\end{aligned}$$

(3) First we'll check $a_n = 2^n - 3^n$ is a solution.

$$\begin{aligned}5a_{n-1} - 6a_{n-2} &= 5(2^{n-1} - 3^{n-1}) - 6(2^{n-2} - 3^{n-2}) \\&= 5\left(\frac{2^n}{2} - \frac{3^n}{3}\right) - 6\left(\frac{2^n}{4} - \frac{3^n}{9}\right) \\&= \left(\frac{5}{2} - \frac{6}{4}\right) 2^n + \left(-\frac{5}{3} + \frac{6}{9}\right) 3^n \\&= \left(\frac{4}{4}\right) 2^n + \left(-\frac{1}{9}\right) 3^n \\&= 2^n - 3^n = a_n \checkmark\end{aligned}$$

Next we'll check $a_n = 2^n$ is a solution.

$$\begin{aligned}5a_{n-1} - 6a_{n-2} &= 5 \cdot 2^{n-1} - 6 \cdot 2^{n-2} \\ &= \frac{5}{2} \cdot 2^n - \frac{6}{4} \cdot 2^n \\ &= \left(\frac{10}{4} - \frac{6}{4}\right) 2^n \\ &= 2^n \\ &= a_n \checkmark\end{aligned}$$

$$(4) \quad 2a_n = a_{n-1} + 2a_{n-2} + n$$

$$\begin{aligned}\text{RHS: } &(A(n-1) + B) + 2(A(n-2) + B) + n \\ &= (An - A + B) + 2(An - 2A + B) + n \\ &= \underline{An} - A + B + \underline{2An} - 4A + 2B + n \\ &= (A + 2A + 1)n + (-A + B - 4A + 2B) \\ &= (3A + 1)n + (3B - 5A)\end{aligned}$$

$$\text{LHS: } 2(A_n + B) = (2A)n + 2B$$

$$\Rightarrow \text{We need } \begin{cases} 3A + 1 = 2A \\ 3B - 5A = 2B \end{cases}$$

So we see that:

$$\begin{aligned}3A + 1 &= 2A & 3B - 5A &= 2B \\ \Leftrightarrow \boxed{A = -1} & & \Leftrightarrow B = 5A = 5(-1) \\ & & \boxed{B = -5}\end{aligned}$$

(5) order = how far relation goes back
homogeneous = no extra constants
linear = all a_i terms linear
constant coefficients = coefficients in front of
 a_i constants

(a) $a_n = a_{n-1} - a_{n-2}^2$
order 2, homogeneous, not linear,
constant coefficients

(b) $a_n = n a_{n-3}$
order 3, homogeneous, linear,
does not have constant coefficients

(c) $a_n = a_{n-2}^3 - a_{n-4} + 3$
order 4, not homogeneous, not linear,
constant coefficients

(d) $a_n = 2a_{n-1} - a_{n-2}$
order 2, homogeneous, linear,
constant coefficients

(e) $a_n = a_{n-1} + 1$
order 1, not homogeneous, linear,
constant coefficients